TRIANGLES

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Introduction

- We all know that triangle is a closed figure which has three sides, three angles and three vertices.
- By name, “Triangle” consists of two words “tri” (which means three) and “angle”. Thus, by name, triangle means something which has three angles.
- For example, in the given triangle ABC (Fig. 1.1):
  - AB, BC, CA are three sides
  - $\angle A$, $\angle B$, $\angle C$ are three angles
  - A, B, C are three vertices

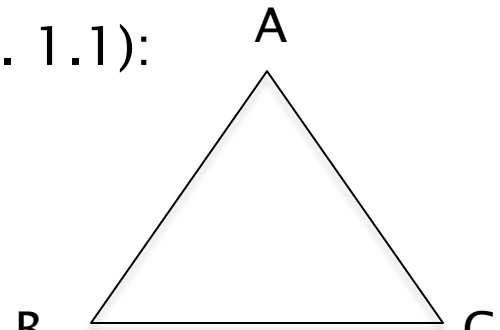


Fig. 1.1

Introduction (Contd..)

- In this presentation, you will study about the following:
  - Congruence of triangles – Meaning of congruence
  - Criteria for congruence of triangles – Congruence rules by which two triangles can proved to be congruent
  - Some properties of a triangle – Theorem of angles opposite to equal sides and converse of this theorem
  - Inequalities in a triangle – Theorems related to side opposite to larger angle and sum of two sides of a triangle in comparison to the third side

Congruence of Triangles

- When two figures are identical in size and shape such that both figures can overlap each other completely, those figures are congruent.
- Some real life examples of congruent figures are two copies of a photograph or two bangles of same size or two one rupee coins of equal size.
- Similarly, when two triangles are congruent if their corresponding sides and corresponding angles are equal.
- Also note that corresponding parts of congruent triangles are equal (denoted as ‘CPCT’ in short).

Congruence of Triangles (Contd..)

- For example, in the given figure (Fig. 1.2), $\triangle ABC$ is congruent to $\triangle A_1B_1C_1$ as they both have equal corresponding sides and equal corresponding angles.

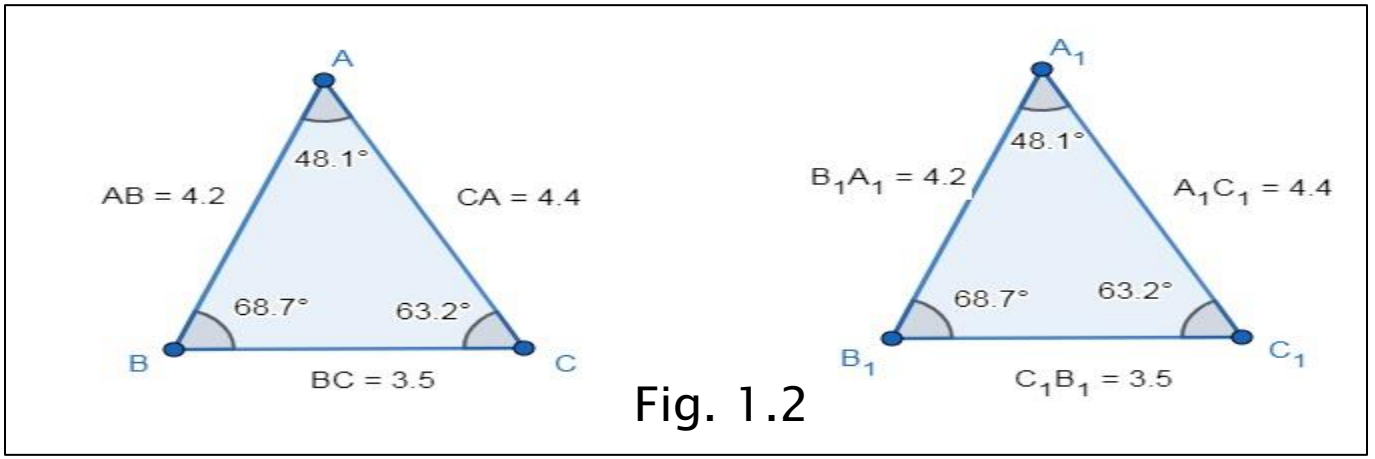


Fig. 1.2

Criteria for Congruence of Triangles

- You will learn about following criterias for Congruence of triangles:
  i. SSS congruence rule (Side-Side-Side)
  ii. SAS congruence rule (Side-Angle-Side)
  iii. ASA congruence rule (Angle-Side-Angle)
  iv. RHS congruence rule (Right Angle-Hypotenuse-Side)

Criteria for Congruence of Triangles (Contd..)

- **SSS congruence rule** says that if three sides of a triangle are equal to the three sides of another triangle, then the two triangles are congruent.
- E.g. In Fig. 1.3, $\triangle ABC$ is congruent to $\triangle DEF$ since $AB=DE=5.4$, $BC=EF=3.5$ and $AC=DF=5$ by SSS congruence rule.

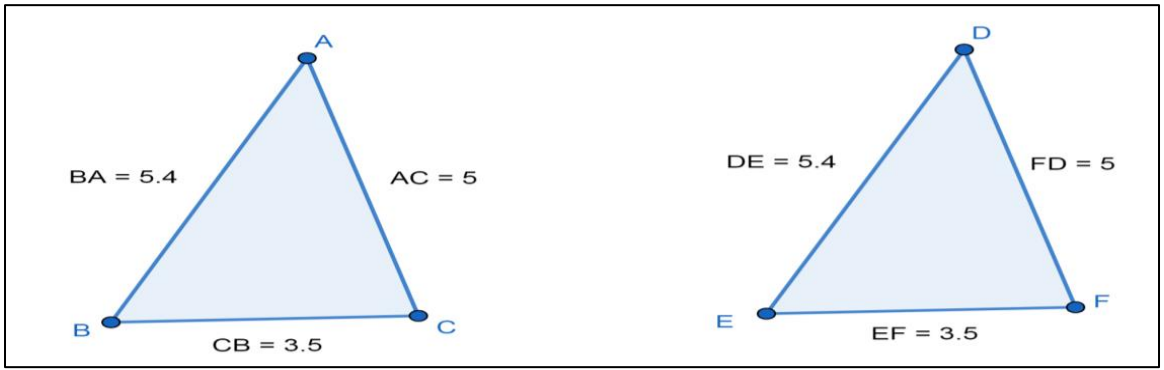
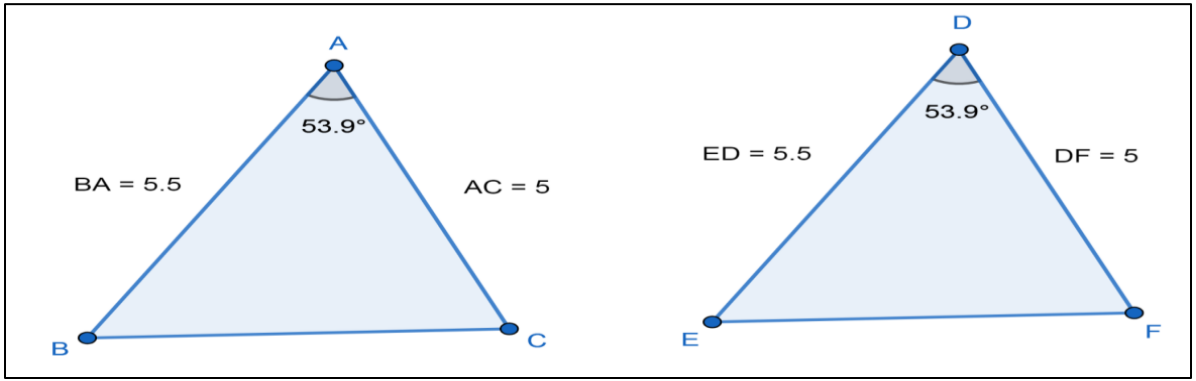


Fig. 1.3

Criteria for Congruence of Triangles (Contd..)

- **SAS congruence rule** says that if two sides and the included angle of a triangle are equal to the two sides and the included angle of the other triangle, then the two triangles are congruent.
- E.g. In Fig. 1.4, $\triangle ABC$ is congruent to $\triangle DEF$ since $AB=DE=5.5$, $\angle A=\angle D=53.9°$ and $AC=DF=5$ by SAS congruence rule.

Fig. 1.4



Criteria for Congruence of Triangles (Contd..)

- **ASA congruence rule** says that if two angles and the included side of a triangle are equal to the two angles and the included side of the other triangle, then the two triangles are congruent.
- E.g. In Fig. 1.5, $\triangle ABC$ is congruent to $\triangle DEF$ since $\angle B = \angle E = 57.3°$, $BC = EF = 5.8$ and $\angle C = \angle F = 60.8°$ by ASA congruence rule.

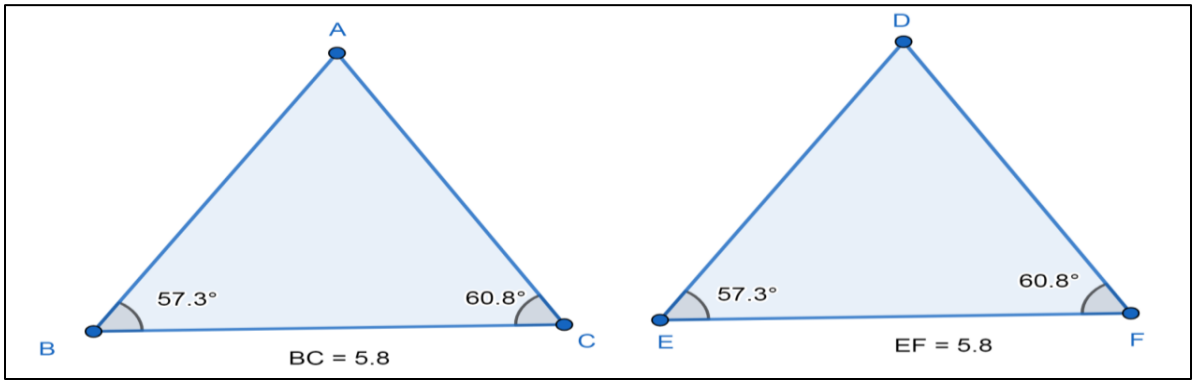
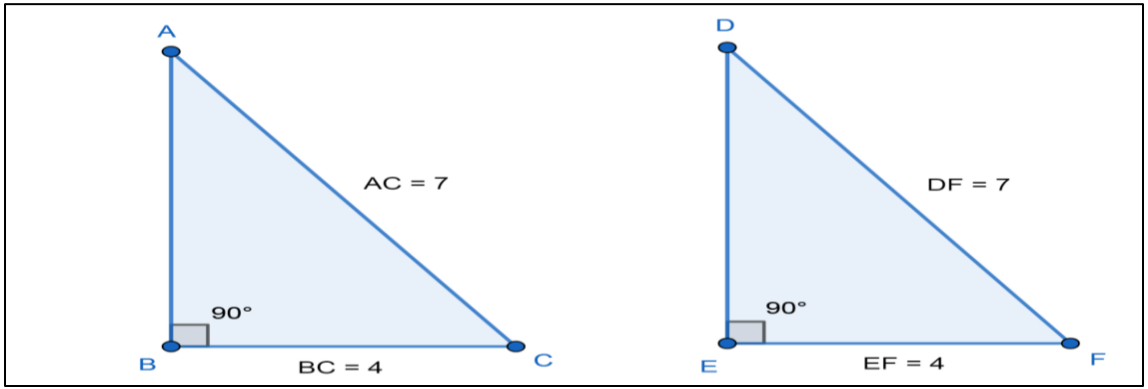


Fig. 1.5

Criteria for Congruence of Triangles (Contd..)

- **RHS congruence rule** says that if in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.
- E.g. In Fig. 1.6, $\triangle ABC$ is congruent to $\triangle DEF$ since $\angle B = \angle E = 90°$, $AC = DF = 7$ and $BC = EF = 4$ by RHS congruence rule.

Fig. 1.6



Some Properties of a Triangle

Property 1: Angles opposite to equal sides of an isosceles triangle are equal.

E.g. In Fig. 1.7, in $\triangle ABD$, since $AD = DB = 5$ then $\angle A = \angle B$ because of Property 1.

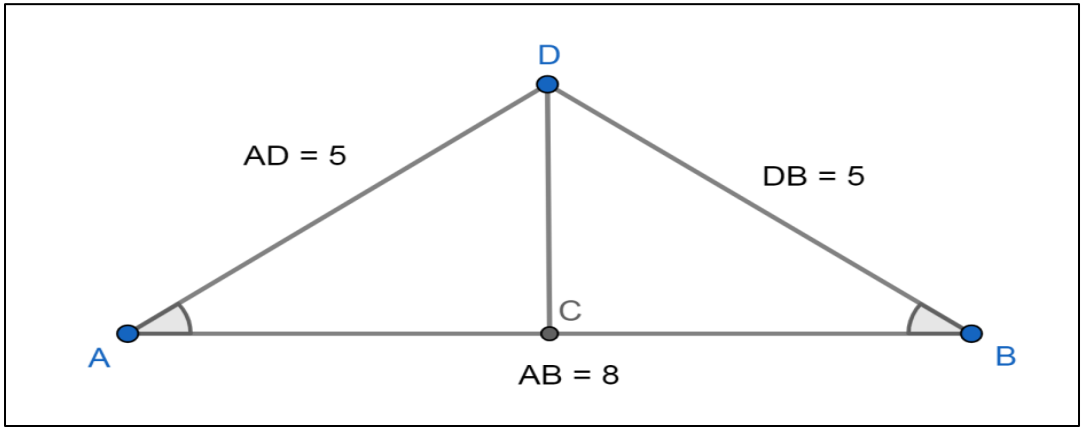


Fig.1.7

Some Properties of a Triangle (Contd..)

Property 2 (Converse of Property 1): The sides opposite to equal angles of a triangle are equal.

E.g. In Fig. 1.8, in $\triangle ABC$, since $\angle A = \angle B = 36.7°$ then $AC = BC$ because of Property 2.

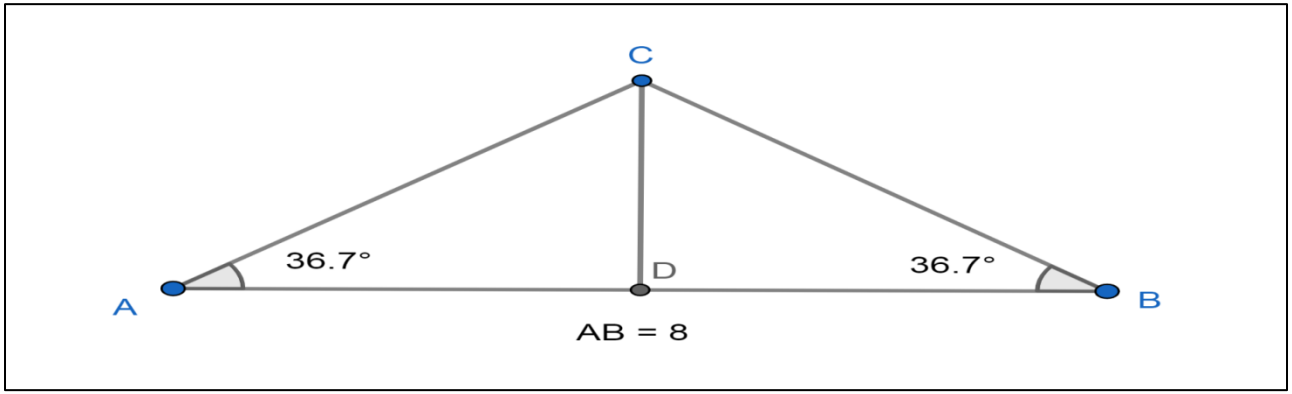


Fig.1.8

Inequalities in a Triangle

Property 3: If two sides of a triangle are unequal, the angle opposite to the longer side is larger.

E.g. In Fig. 1.9, AB=8 and BC=6 i.e. AB>BC.
Angle opposite to AB=∠C,
Angle opposite to BC=∠A

Using Property 3, since AB>BC, so ∠C>∠A.

We can see that ∠C=100.9° and ∠A=47.1°

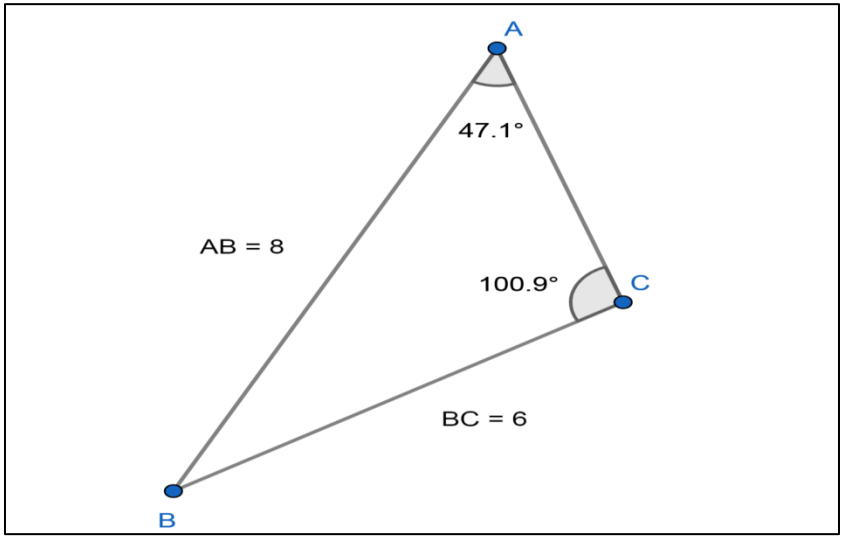


Fig. 1.9

Inequalities in a Triangle (Contd..)

Property 4: In any triangle, the side opposite to the larger angle is longer.

E.g. In Fig. 1.10, $\angle B = 72.6°$ and $\angle A = 21.3°$ i.e. $\angle B > \angle A$.
Side opposite to $\angle B = AC$,
Side opposite to $\angle A = BC$

Using Property 4, since $\angle B > \angle A$, so $AC > BC$.

We can see that $\angle C = 100.9°$ and $\angle A = 47.1°$

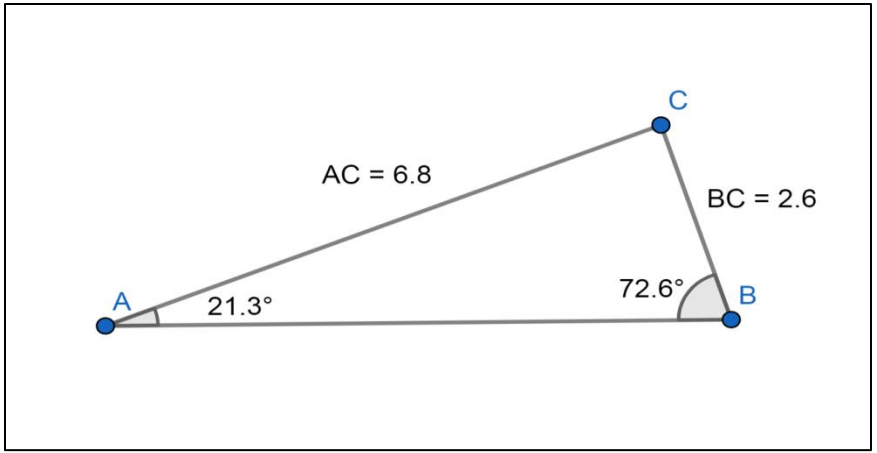


Fig. 1.10

Inequalities in a Triangle (Contd..)

Property 5: The sum of any two sides of a triangle is greater than the third side.

E.g. In Fig. 1.11, AC=5, AB=6 and BC=7

Here, AC+AB=5+6=11

AC+BC=5+7=12

AB+BC=6+7=13

We can see that, AC+AB>BC

AC+BC>AB

AB+BC>AC

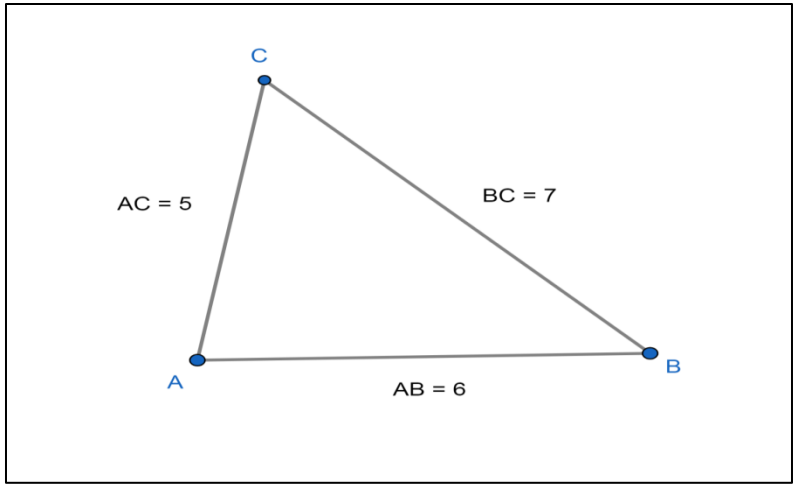


Fig. 1.11

Summary

- Two triangles are congruent is their corresponding sides and angles are equal.
- Corresponding parts of congruent triangles are equal.
- Four criterias for congruence of triangles are:
  i. SSS (Side-Side-Side) – If three corresponding sides are equal
  ii. SAS (Side-Angle-Side) – If two sides and included angle are equal
  iii. ASA (Angle-Side-Angle) – If two angles and included side are equal
  iv. RHS (Right Angle-Hypotenuse-Side) – If in two right angled triangles, hypotenuse and one other side are equal

Summary (Contd.)

- Some Properties of a Triangle are:
  - Angles opposite to equal sides of an isosceles triangle are equal
  - Sides opposite to equal angles of a triangle are equal.

- Properties relating to inequalities in a triangle are:
  - If two sides of a triangle are unequal, the angle opposite to longer side is larger.
  - In any triangle, the side opposite to the larger angle is longer.
  - The sum of any two sides of a triangle is greater than the third side.

THANK YOU